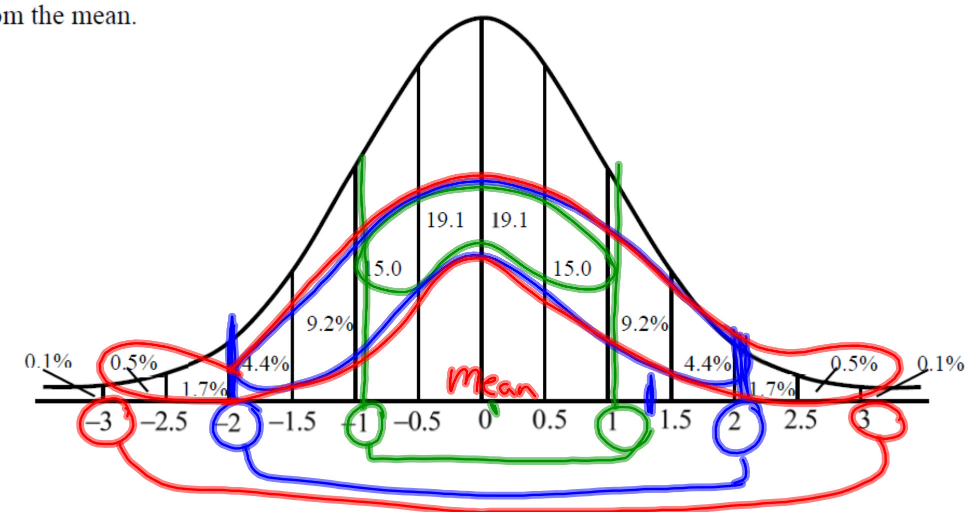
6/1/17 "Live as if you were to die tomorrow. Learn as if you were to live forever."
-Mahatma Gandhi

HW: "Normal Distribution and Z-scores" homework section
Test 3 Wednesday 6/7

AIM: What is the Normal Curve?

Warm Up:

Many populations have a distribution that can be well described with what is known as **The Normal Distribution** or the **Bell Curve**. This curve, shows the percent or proportion of a normally distributed data set that lies certain amounts from the mean.



***Exercise* #1:** For a population that is normally distributed, find the percentage of the population that lies

(a) within one standard deviation of the mean.

15 + 19.1 + 19.1 + 15 = 68.2%

(b) within two standard deviations of the mean.

4.4 + 9.2 + 68.2 + 9.2 + 4.4 =
95.4%

(c) within three standard deviations of the mean.

99.8%

The Empirical Rule:

68% within one std deviation
95% within 2 std deviations
99% within 3 std deviations

As can be easily seen from *Exercise* #1, the majority of any normally distributed population will lie within one standard deviation of its mean and the vast majority will lie within two standard deviations. A whole variety of problems can be solved if we know that a population is normally distributed. The normal distribution can be used in increments other than half-standard deviations. In fact, we can use our calculators to determine probabilities (or proportions) for almost any data value within a normally distributed population, as long as we know the population mean, μ, and the population standard deviation, σ. But, first, we will introduce a concept known as a data value's z-score.

THE Z-SCORE OF A DATA VALUE

For a data point x_i, its z-score is calculated by: $z = \dfrac{x_i - \mu}{\sigma}$. It calculates how far from the mean, in terms of standard deviations, a data point lies. It can be positive if the data point lies above the mean or negative if the data point lies below the mean.

$$z = \frac{\text{observed score} - \text{mean}}{\text{standard deviation}}$$

Exercise #2: ~~Boy's heights in~~ seventh grade are normally distributed with a mean height of 62 inches and a standard deviation of 3.2 inches. Find z-scores, rounded to the nearest hundredth, for each of the following heights. Show the calculation that leads to your answer.

(a) $x_i = 66$ inches

$\dfrac{66-62}{3.2} = 1.25$

(b) $x_i = 57$ inches

$\dfrac{57-62}{3.2} = -1.56$

(c) $x_i = 70$ inches

$\dfrac{70-62}{3.2} = 2.50$

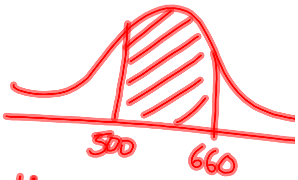
However, our calculator can determine probabilities (or proportions) for almost any data value within a normally distributed population without us needing to calculate z scores at all!

We use a command called normalcdf which can be found under our distribution menu (2nd Vars 2).

normal cdf (lowest, highest, mean, standard deviation)

Exercise #3: At Arlington High School, 424 juniors recently took the SAT exam. On the math portion of the exam, the mean score was 540 with a standard deviation of 80. If the scores on the exam were normally distributed, answer the following questions.

(a) What percent of the math scores fall between 500 and 660?



NORMAL FLOAT DEC REAL RADIAN MP
normalcdf
lower: 500
upper: 660
μ:0 540
σ:1 80
Paste

μ = mean σ = std dev.

≈ 62.5%

(b) How many scores fell between 500 and 660? Round your answer to the nearest whole number.

People times %

424 × .625

≈ $\boxed{265 \text{ scores}}$

This process is sometimes used to determine a particular data point's **percentile**, which is the **percent of the population equal to or less than the data point**.

(c) If Evin scored a 740 on her math exam, what percent of the students who took the exam did better than her?

NORMAL FLOAT DEC REAL RADIAN MP
normalcdf(740,99999,540,8▸
.0062096799

big #

.6%

(d) Approximately how many students did better than Evin?

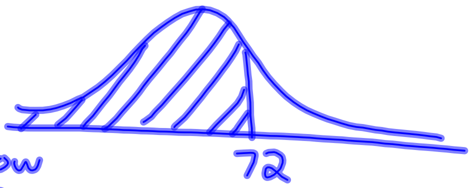
424 × .006

≈ $\boxed{3 \text{ students}}$

Exercise #4: The heights of 16 year old teenage boys are normally distributed with a mean of 66 inches and a standard deviation of 3. If Jabari is 72 inches tall, which of the following is closest to his height's percentile rank?

(1) 85th (3) 98th

(2) 67th (4) 93rd



−99999

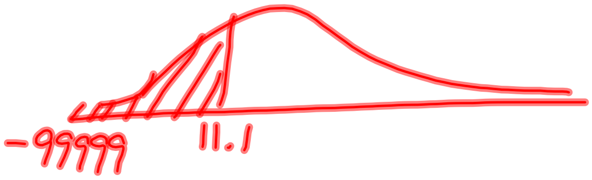
NORMAL FLOAT DEC REAL RADIAN MP
normalcdf(-99999,72,66,3)
.977249938

≈ 98th percentile

***Exercise* #5:** The amount of soda in a standard can is normally distributed with a mean of 12 ounces and a standard deviation of 0.6 ounces. If 250 soda cans were pulled by a company to check volume, how many would be expected to have less than 11.1 ounces in them?

(1) 17 (3) 28

(2) 23 (4) 11



$\approx 6.68\%$

$250 \times .0668 \approx 17$

***Exercise* #6:** Biologists are studying the weights of Red King Crabs in the Alaskan waters. They sample 16 crabs and compiled their weights, in pounds, as shown below.

9.8, 10.1, 11.1, 12.4, 11.8, 13.2, 12.8, 12.5, 13.7, 11.6, 13.4, 12.3, 12.6, 14.8, 14.2 15.1

(a) Determine the mean and sample standard deviation for this sample of crabs. Round both statistical measures to the nearest *tenth* of a pound.

$\bar{x} = 12.6$

$S_x = 1.5$

(b) Why does this sample indicate that the population would be well modeled using a normal distribution? Explain. Hint – Use your calculator to sort this data in ascending order.

$12.6 + 1.5 = 14.1$

$12.6 - 1.5 = 11.1$

$\frac{11}{16} = 68\%$

✓

(c) Assuming your mean and standard deviation from part (a) apply to a normally distributed population of crabs caught in Alaska, what percent will fall between 9.6 pounds and 15.6 pounds?

normal cdf(9.6, 15.6, 12.6, 1.5)

$\approx 95.4\%$

(d) If fishermen must throw back any crab caught below 10.4 pounds, approximately what percent of the crabs caught will need to be thrown back if the weights are normally distributed?

7.1%

***Exercise* #7:** If the scores on a standardized test are normally distributed with a mean of 560 and a standard deviation of 75. Answer the following questions by using z-scores and the normal distribution table.

(a) Find the probability that a test picked at random would have a score larger than 720. Round to the nearest hundredth of a percent. ← 2ND VARS 2

lowest: 720
high: 99999
$\mu = 560$ = .0164
$\sigma = 75$
$\boxed{1.64\%}$

(b) Find the probability that a completed test picked at random would have a score less than 500. Round to the nearest tenth of a percent.

low: -99999
high: 500
$\mu = 560$ = 21.2%
$\sigma = 75$

(c) Find the probability that a completed test picked at random would have a score between 500 and 600.

low: 500
high: 600
μ: 560 = 49%
$\sigma = 75$

(d) Find the probability that a completed test picked at random would have a score between 600 and 700.

low: 600
high: 700
$\mu = 560$ = 27%
$\sigma = 75$

There is another very helpful calculator command. It can find the data point that is at a specific percentile. It is called invNorm. It can be found under our distribution menu (2nd Vars 3).

(e) What would a student have to score to have scored at the 95th percentile?

percent
average/mean
std deviation

area = .95
$\mu = 560$
$\sigma = 75$ $\approx \boxed{683}$

(f) What score would be at the 15th percentile?

Area = .15
$\mu = 560$
$\sigma = 75$ $\approx \boxed{482}$

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- What Percent/Probability: use normalcdf

  $\boxed{\text{2nd}}$ $\boxed{\text{Vars}}$ $\boxed{2}$

- What score/value is at a percentile: use "InvNorm"

  $\boxed{\text{2nd}}$ $\boxed{\text{Vars}}$ $\boxed{3}$

- To find Mean, median, standard deviation, quartiles

  $\boxed{\text{Stat}}$ $\boxed{>}$ $\boxed{1}$